



# A Survey of Ballistic Transfers to the Lunar Surface

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# Overview

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- Background
- Models
- Planar Analysis
  - CRTBP, Earth-Moon (EM) Ephemeris, Sun-Earth-Moon (SEM) Ephemeris
  - Invariant Manifolds
- Spatial Analysis
  - Earth-Moon, Sun-Earth-Moon Ephemeris
  - Trajectory Characteristics
- Conclusions

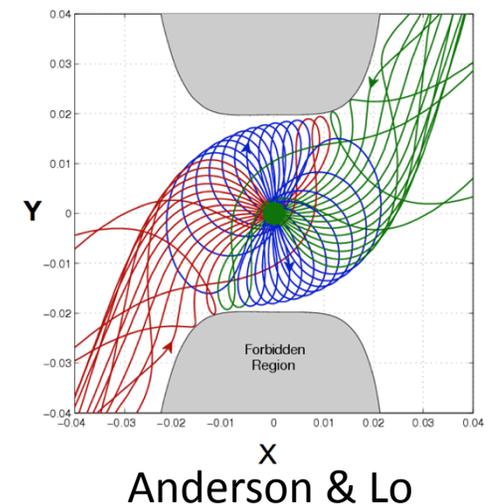
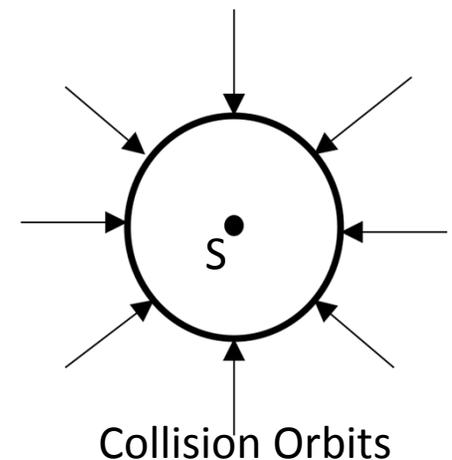
# Transfer Types

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- Direct Transfers
  - Apollo type transfers
  - Short TOFs (~ 3-6 days)
- Three-Body Low-Energy Transfers
  - Longer TOFs (several days → ∞)
- Four-Body Low-Energy Transfers (Requires Sun's Influence)
  - Japanese Mission Muses-A (Hiten) (Belbruno & Miller, 1993)
  - Koon, Lo, Marsden & Ross (2001): Tools to reproduce Hiten-like trajectory using invariant manifolds
  - Parker & Lo (2005): Sun-Earth-Moon trajectories in 3D
- Parker (2007): Surveyed trajectories to lunar halo orbits

# Collision Orbit Studies

- Easton (1971), McGehee (1974)
  - Collision orbits from mathematical perspective
- Anderson & Lo (2005): Jupiter-Europa analysis
  - Integrate backward in plane & 3D, compute characteristics
  - Examined non-perpendicular cases
- Kirchbach, Zhend, Aristoff, Kavanagh, Villac & Lo (2005)
  - Focused on range of angles for planar escape case
  - Check for Jupiter or Europa impacts
- Applications
  - Impactors ( $\sim$  normal to surface, LCROSS =  $86^\circ$  impact angle)
  - Landers ( $\Delta V$  approx., modify to produce trajectory)



# Models

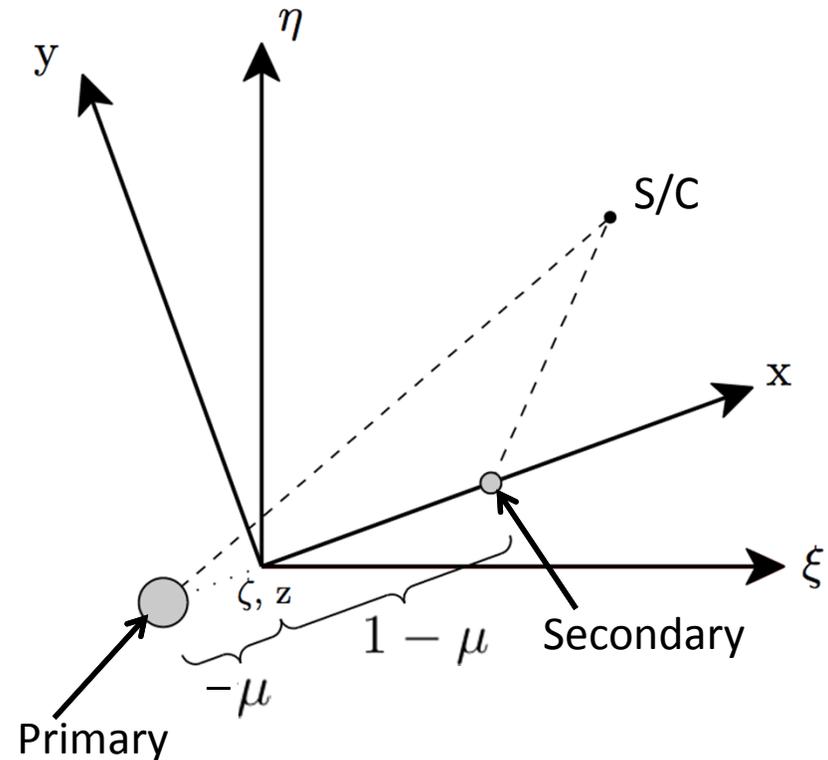
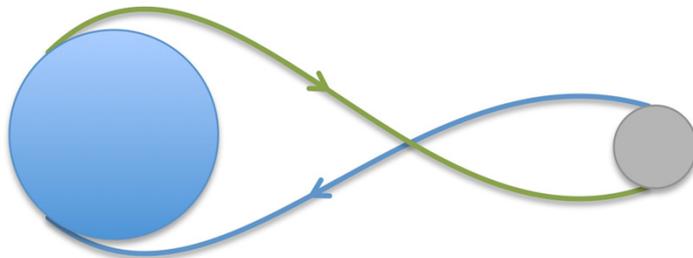
- Circular Restricted Three-Body Problem (CRTBP)

- Five Equilibrium Points ( $L_1, L_2, L_3, L_4, L_5$ )
- Symmetry:

$$(x, y, z, \dot{x}, \dot{y}, \dot{z}, t)$$



$$(x, -y, z, -\dot{x}, \dot{y}, -\dot{z}, -t)$$



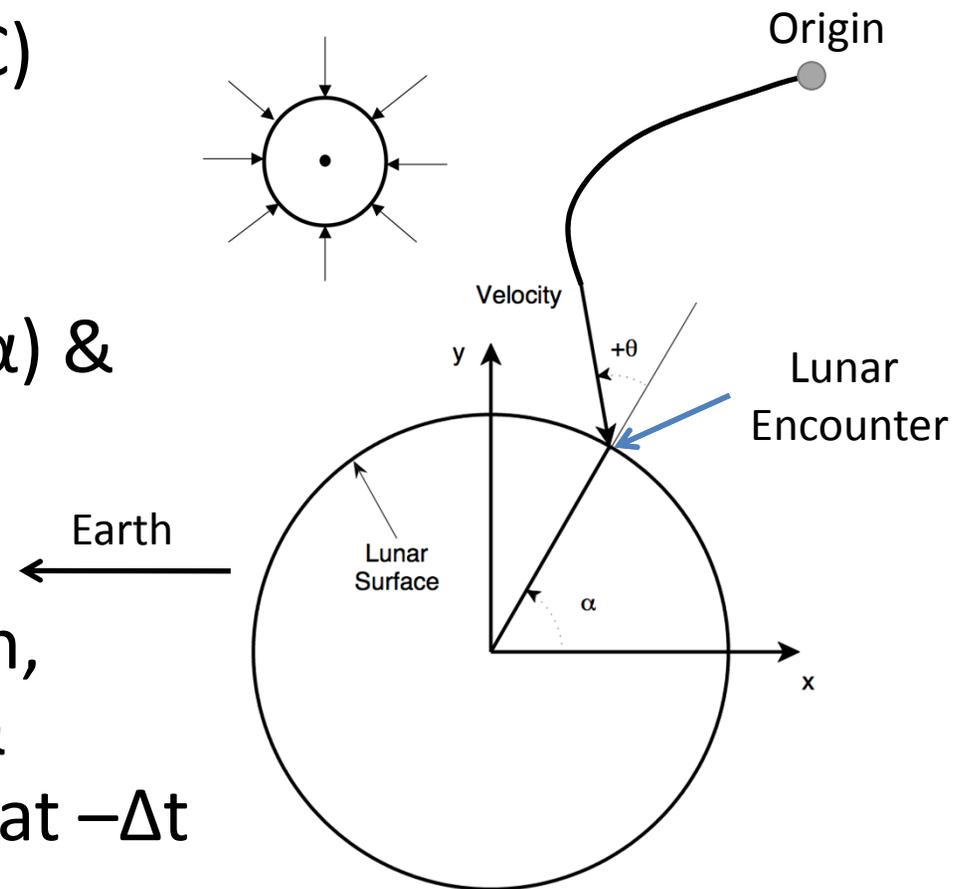
Jacobi Constant:

$$C = x^2 + y^2 + \frac{2(1-\mu)}{r_1} + \frac{2\mu}{r_2} - \dot{x}^2 - \dot{y}^2 - \dot{z}^2$$

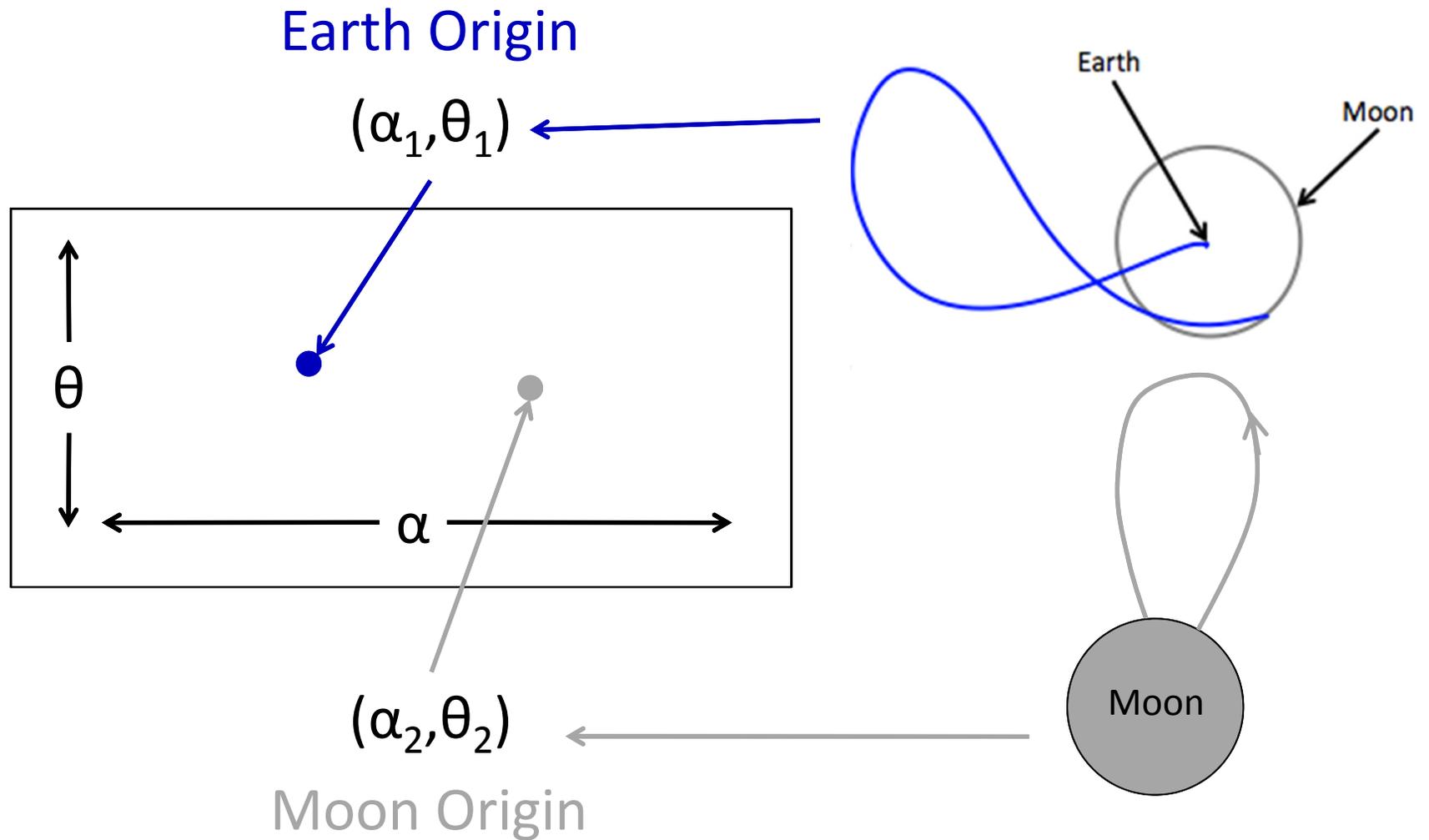
- Ephemeris Model: JPL DE421 (Sun, Earth, Moon)

# Planar Analysis Method

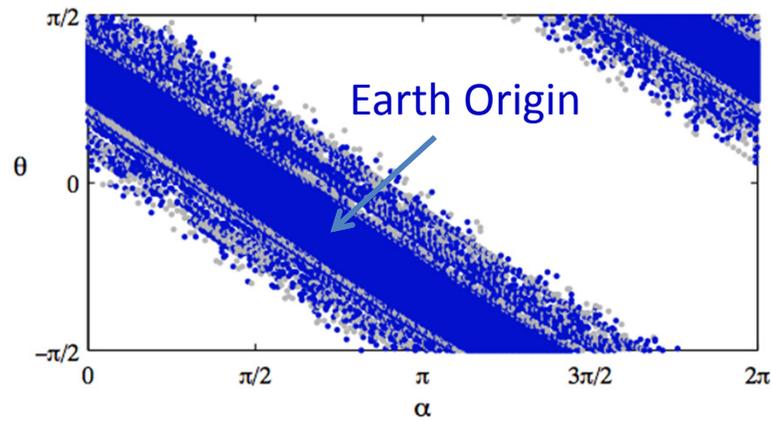
- Vary Jacobi Constant ( $C$ )
- Compute  $|\mathbf{V}| = f(C)$  at surface
- Vary surface location ( $\alpha$ ) & velocity direction ( $\theta$ )
- Integrate for  $-\Delta t$
- Determine origin (Earth, Moon, or elsewhere) & parameters of interest at  $-\Delta t$
- $-\Delta t = -200$  days



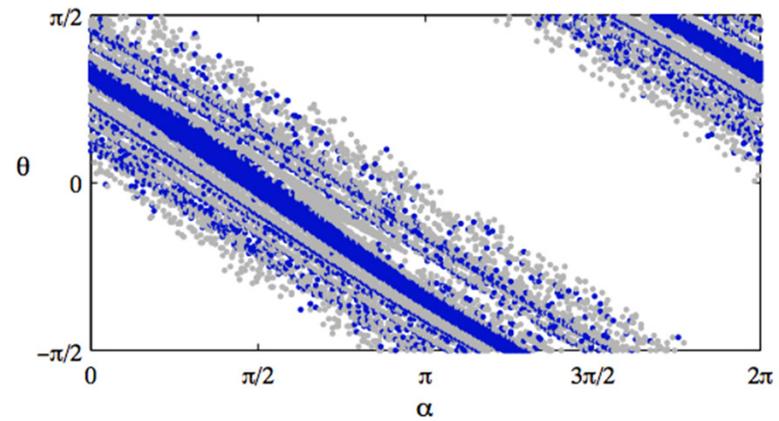
# Plot Origin of Trajectory



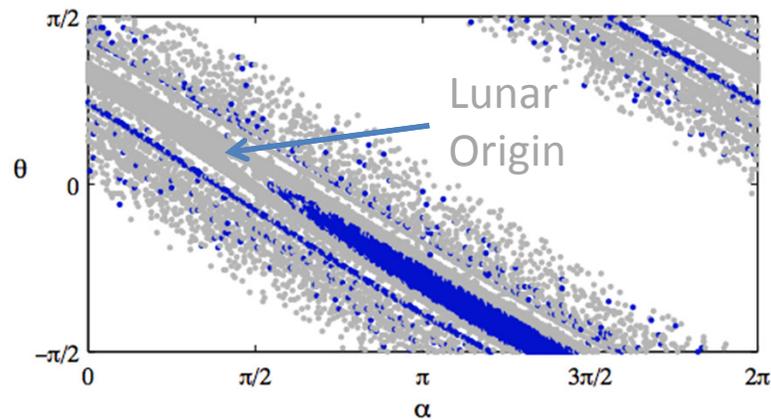
# CRTBP (Earth-Moon)



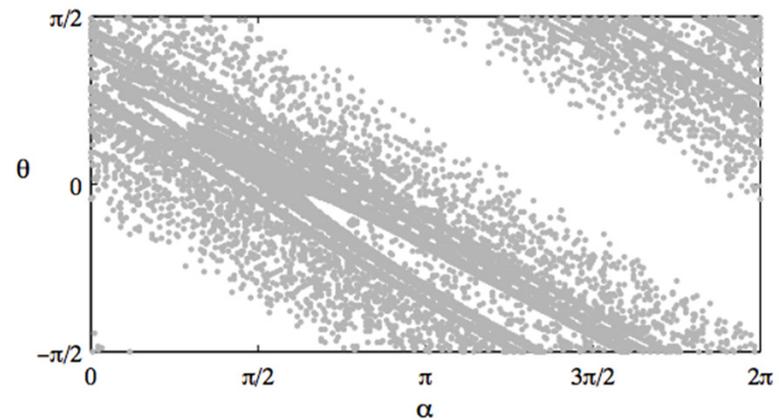
$C = 2.2$



$C = 2.4$



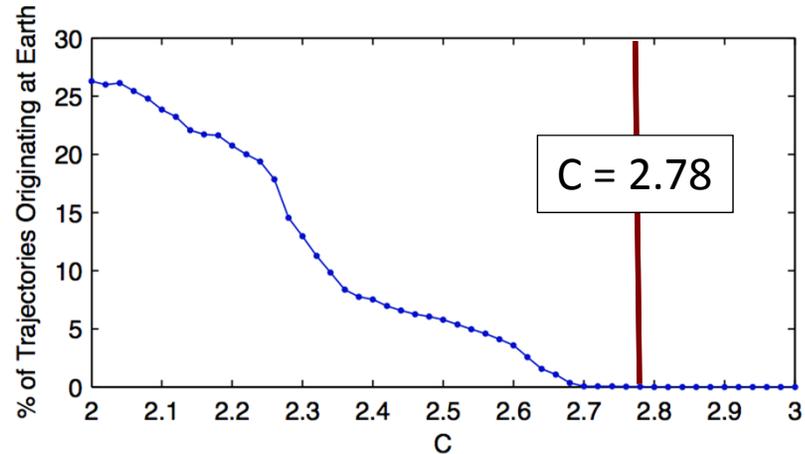
$C = 2.6$



$C = 2.8$

# Earth Origin Trajectories Vary with C

- Over ¼ originate at Earth for low C
- Decreases to 0.03 % percent at C = 2.78

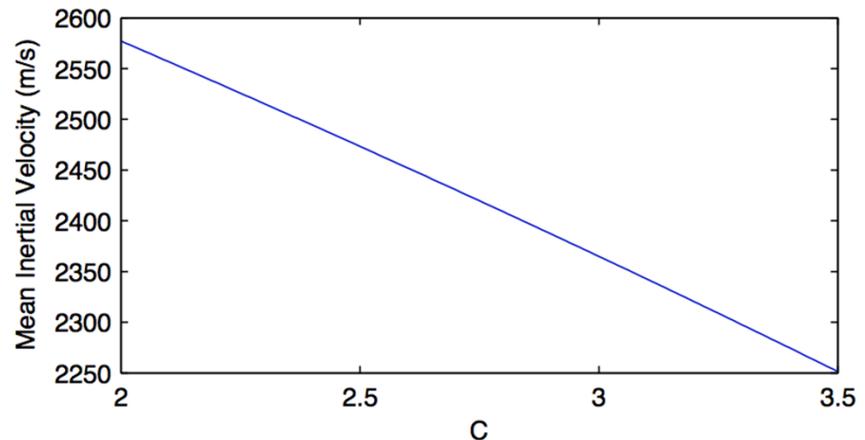


- Mean velocities approximately constant on surface for each C

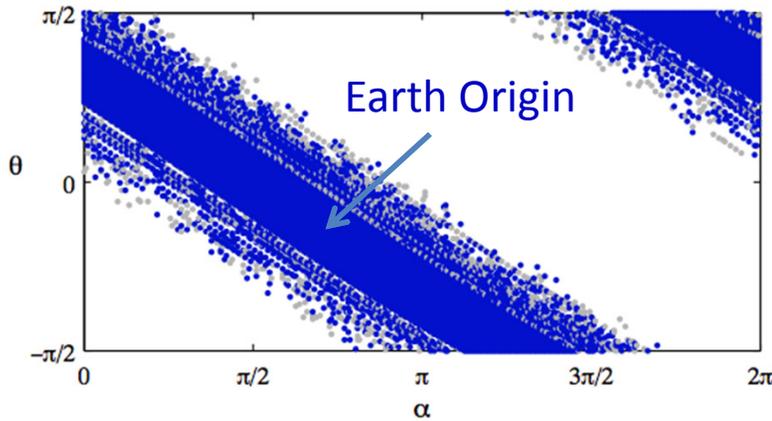
Higher C



Lower Velocity

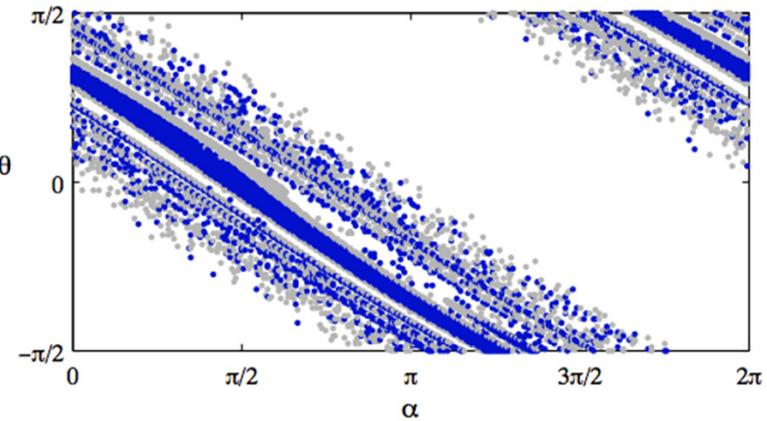


# Earth-Moon Ephemeris

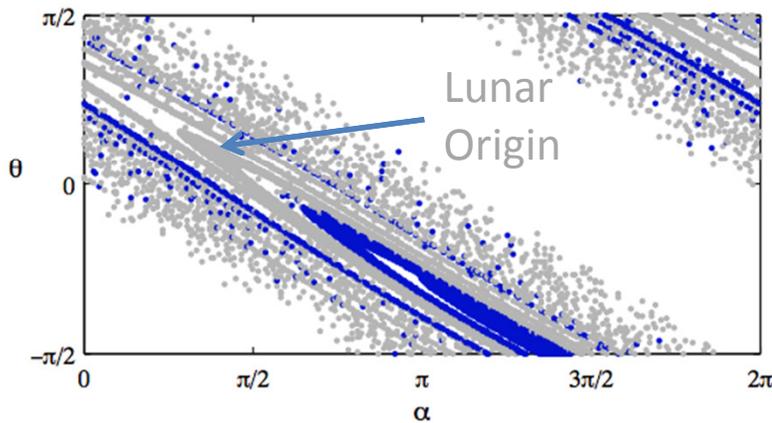


$C = 2.2$

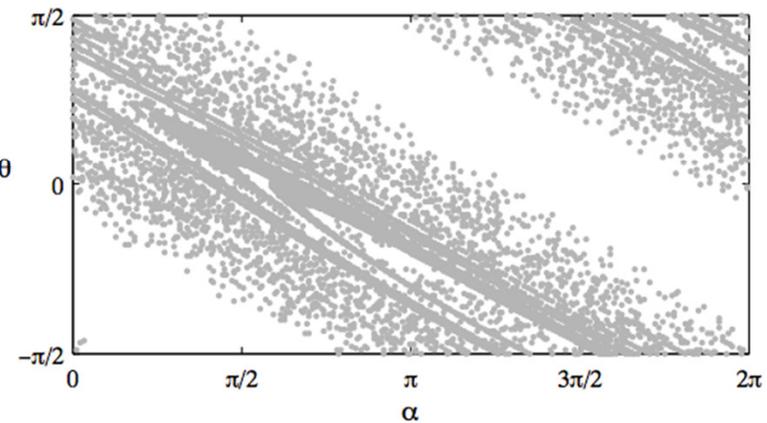
(January 1, 2015)



$C = 2.4$



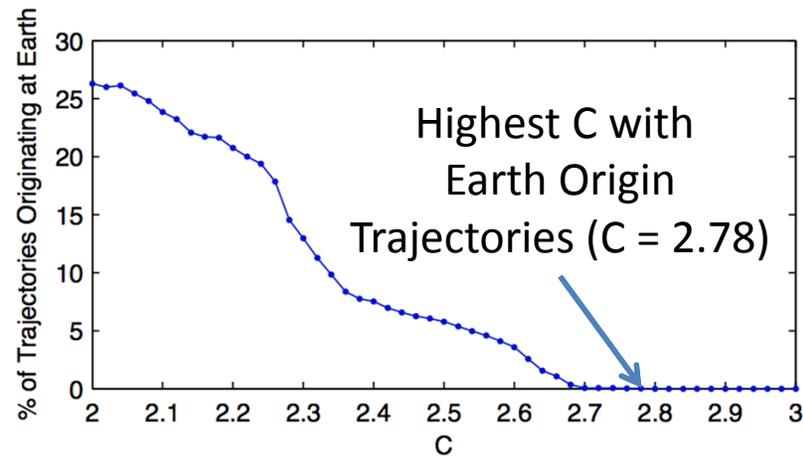
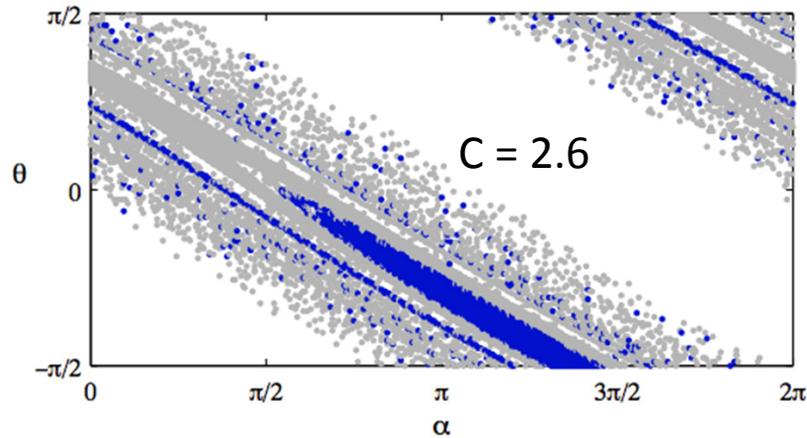
$C = 2.6$



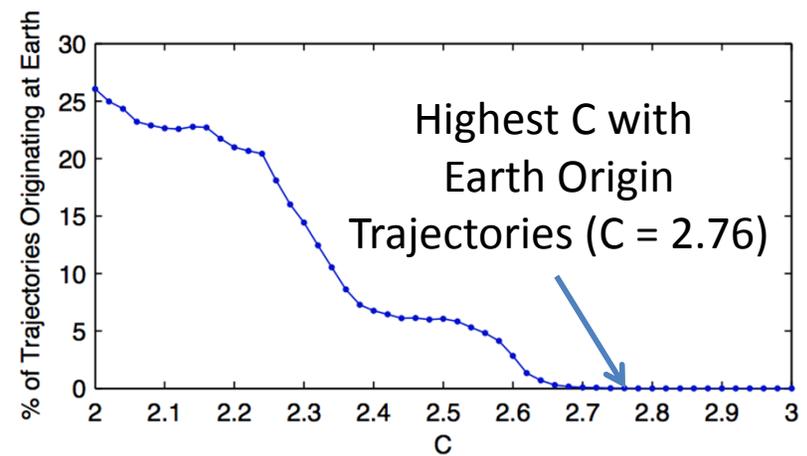
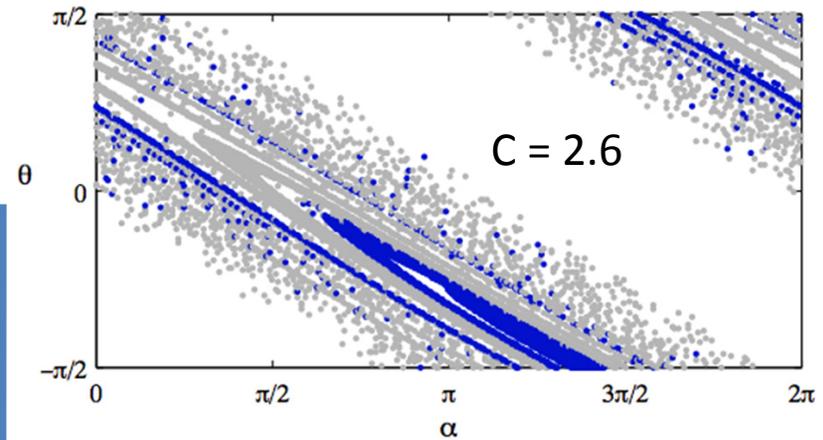
$C = 2.8$

# Model Results Match Closely

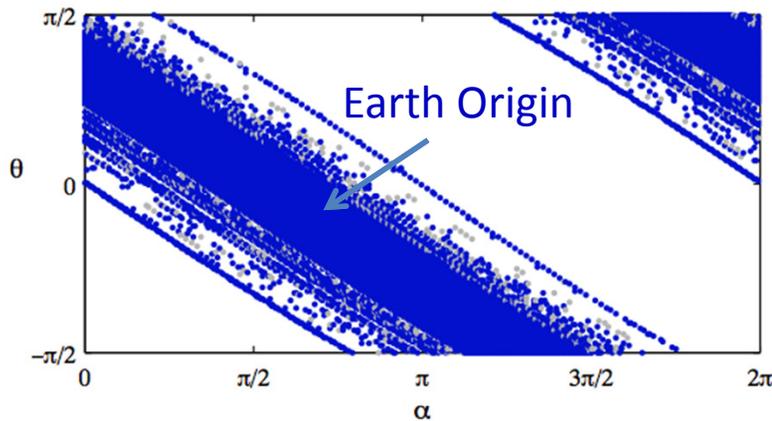
CRTBP



Earth-Moon Ephemeris

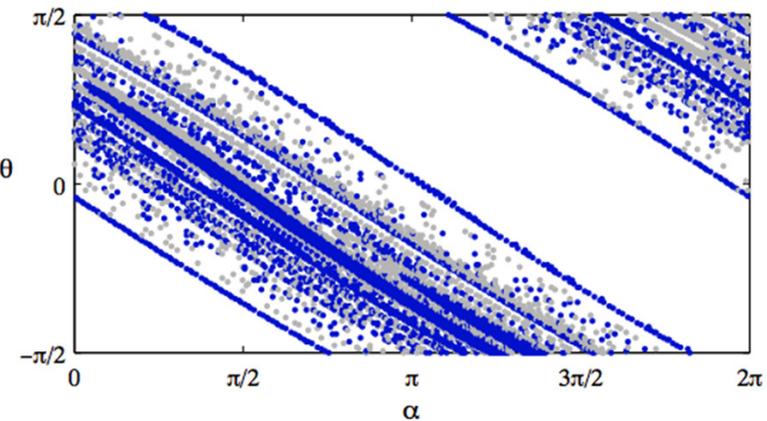


# Sun-Earth-Moon Ephemeris

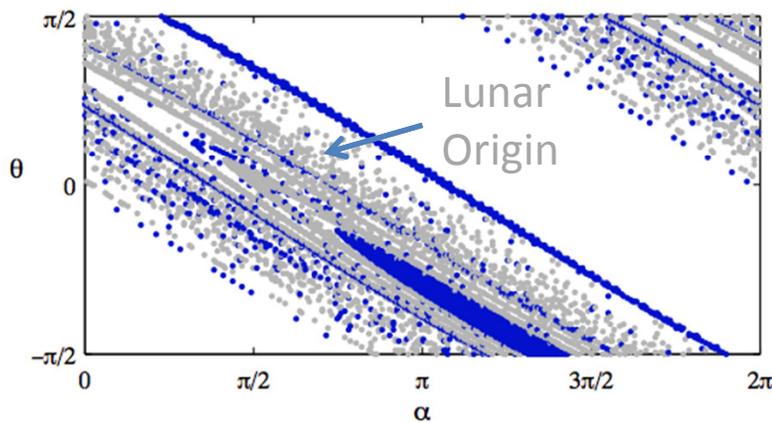


$C = 2.2$

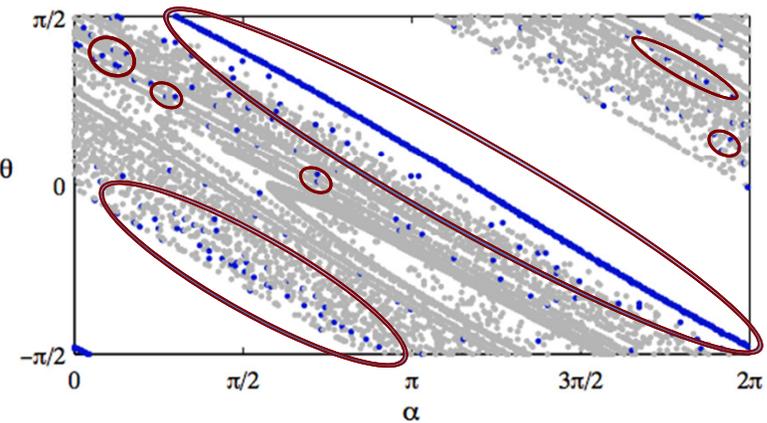
(January 1, 2015)



$C = 2.4$

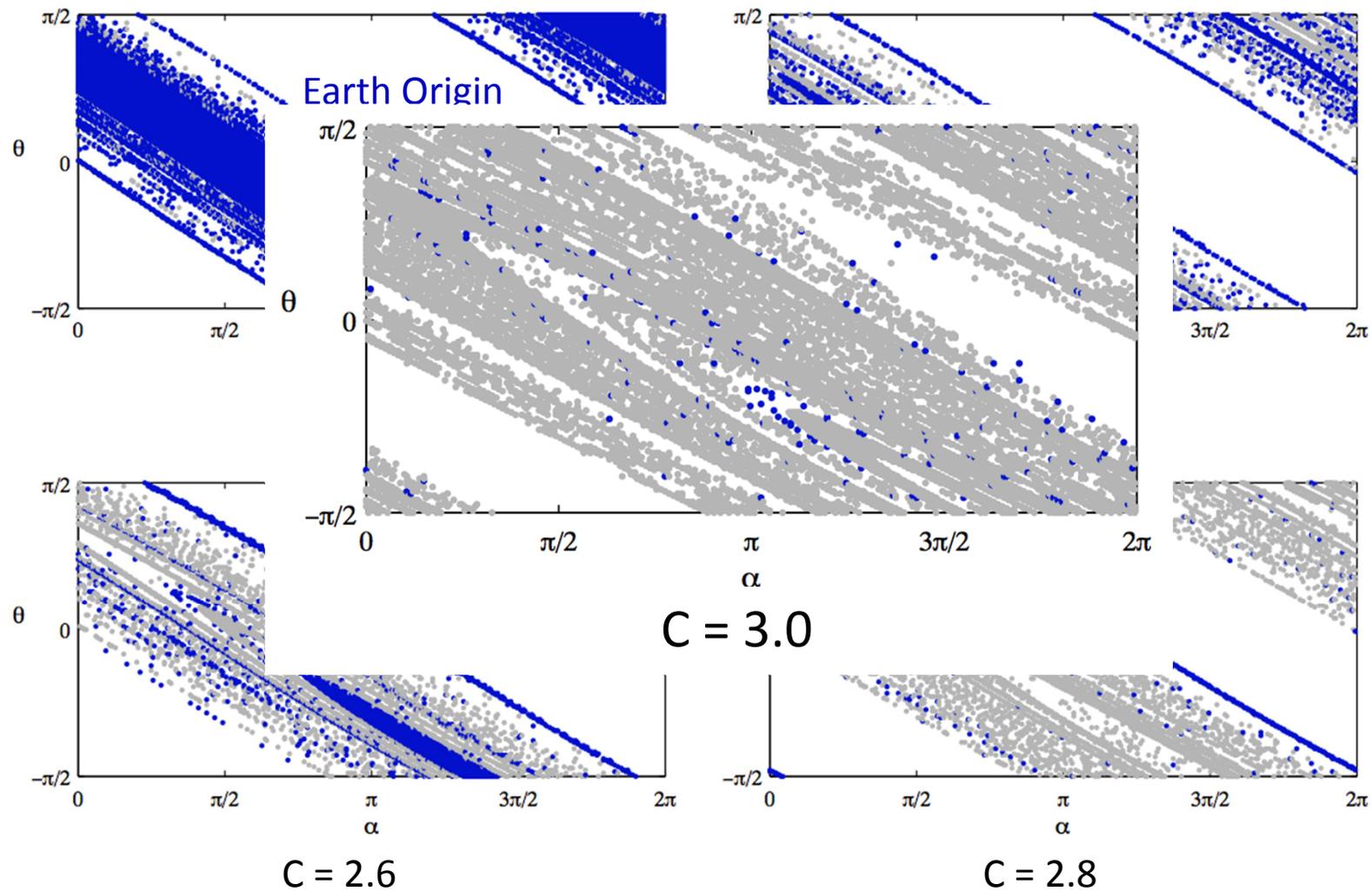


$C = 2.6$

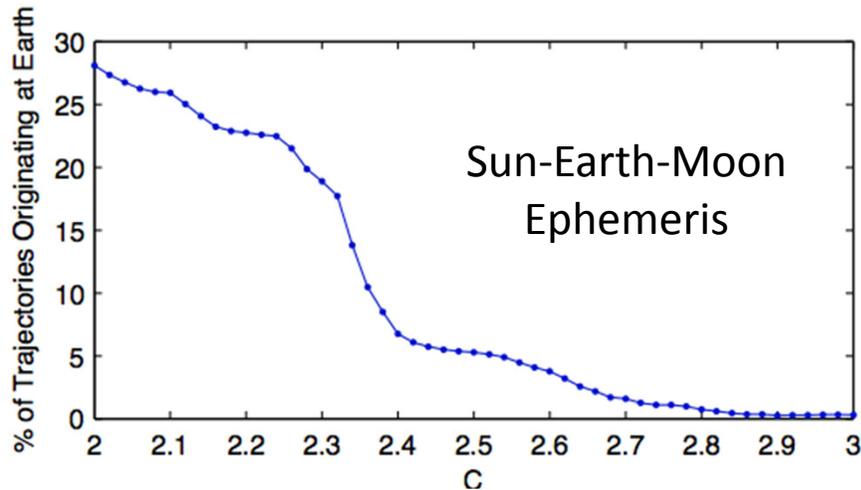


$C = 2.8$

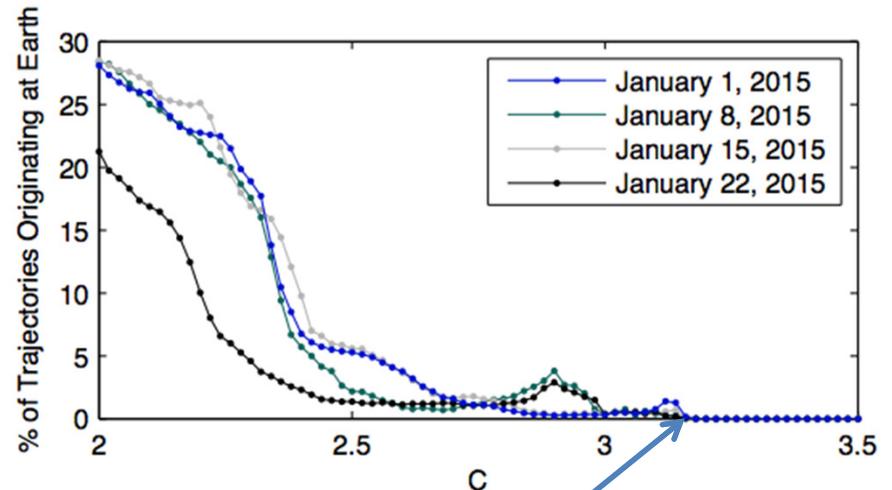
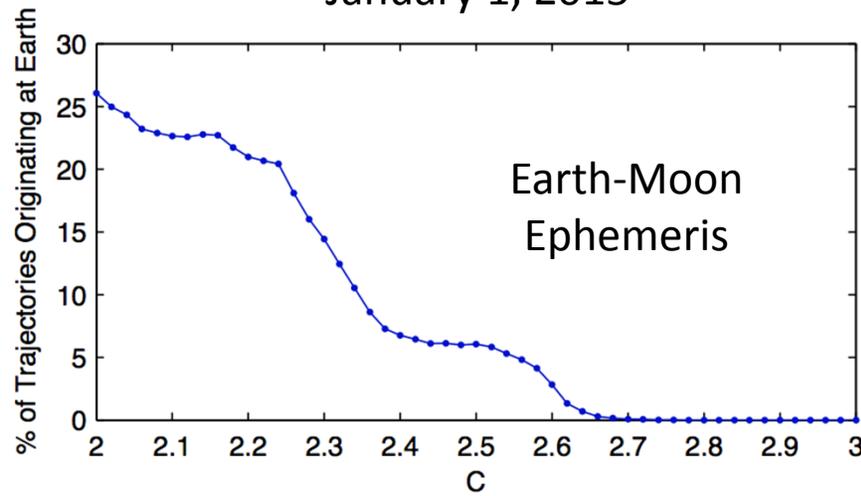
# Earth-Moon Trajectories Still Exist at $C = 3.0$



# Variation with Epoch

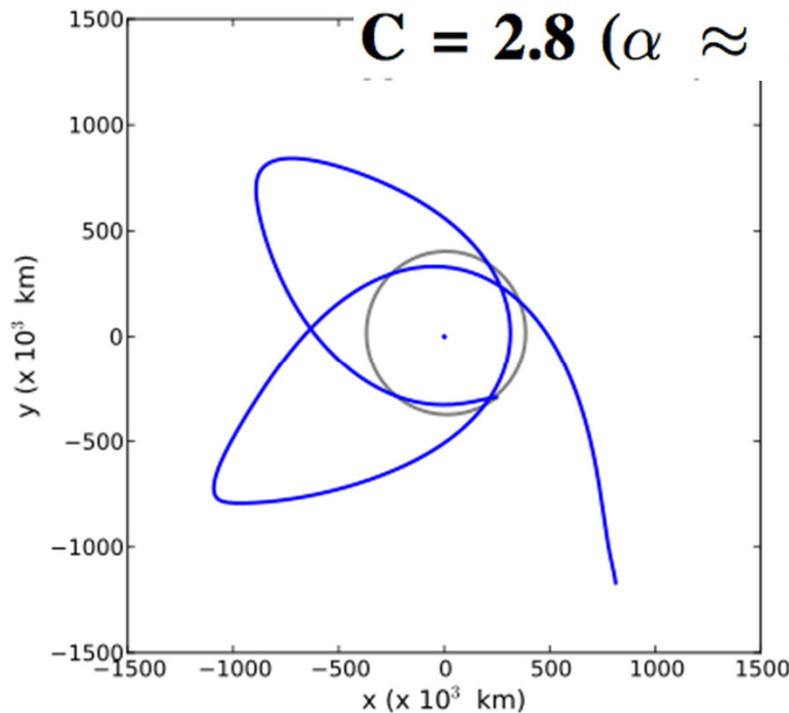


January 1, 2015



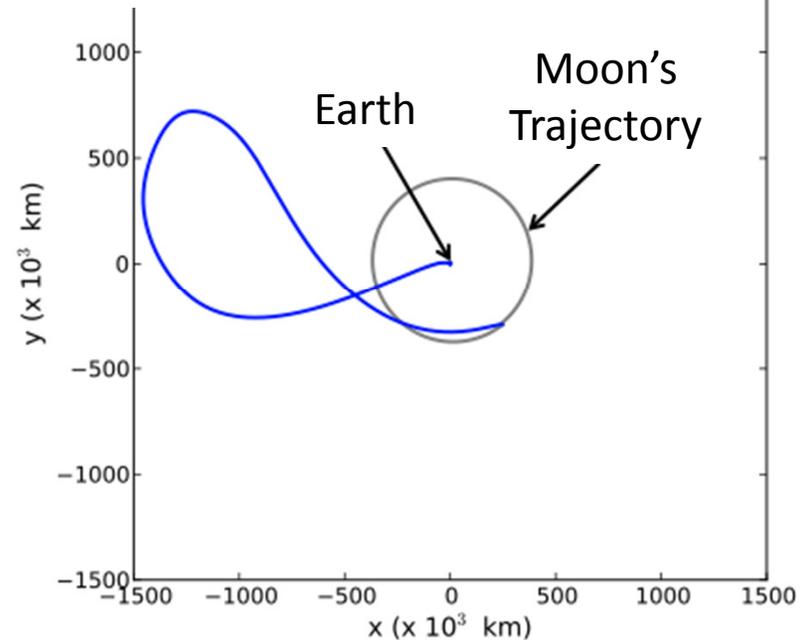
- 0.15 % of trajectories still originate at Earth at  $C = 3.16$
- Significantly higher than Earth-Moon system
- Indicates that lower-energy trajectories may be used, use the Sun

# Difference in EM & SEM Trajectories



Earth-Moon

- Trajectory does not approach Earth
- Final point is beyond the Moon's orbit

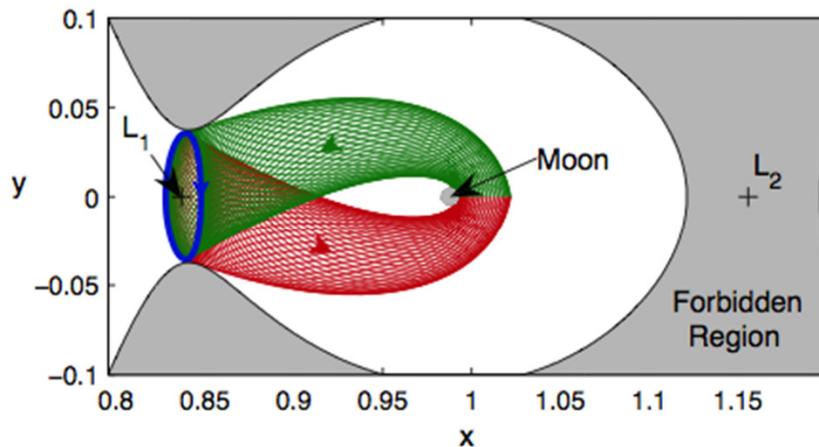


Sun-Earth-Moon

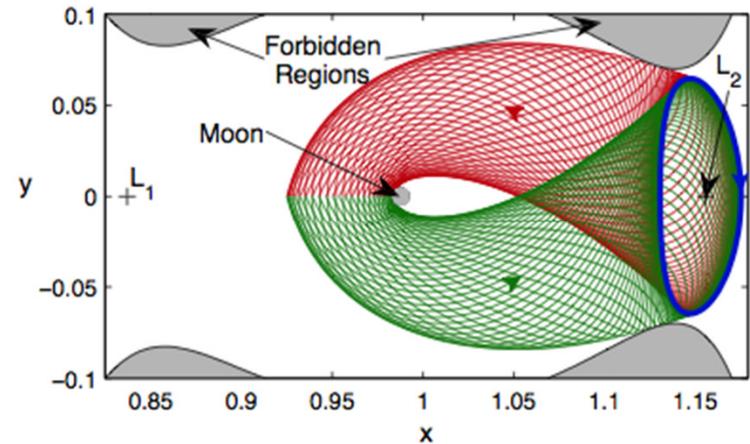
- Travels toward  $L_1$  point
- Heuristically, follows invariant manifolds of libration orbit

# Manifold Comparison

## Libration orbit manifolds for Earth-Moon System



$L_1 : C = 3.18$   
Tangent Velocity = 2298 m/s



$L_2 : C = 3.16$   
Tangent Velocity = 2330 m/s

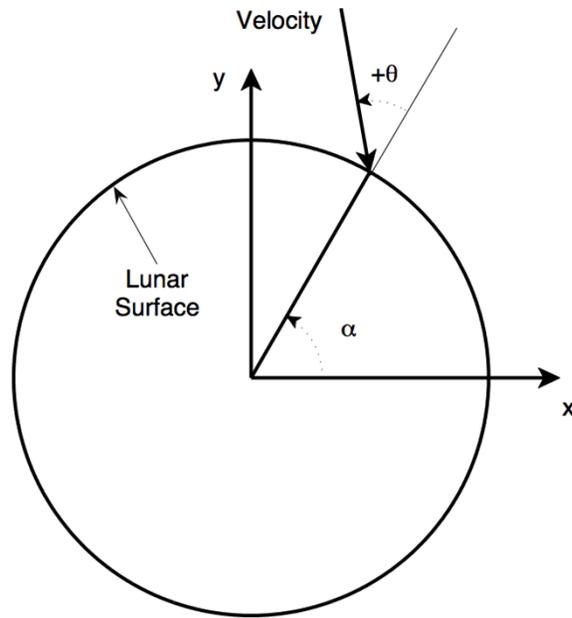
- Velocities similar to other low-energy cases
- Requires connecting Earth & libration orbit

# Planar results

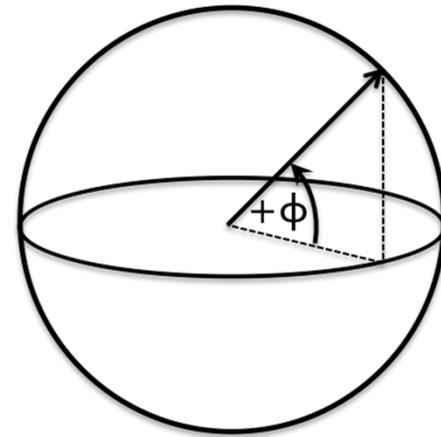
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- Trajectories in CRTBP compare well to Earth-Moon ephemeris model
- Statistically significant set of trajectories that use Sun and
- Sun-influenced trajectories provide lower velocities at Moon (more than point designs found previously)
- These trajectories heuristically appear to follow invariant manifolds in Sun-Earth libration points

# Spatial Analysis

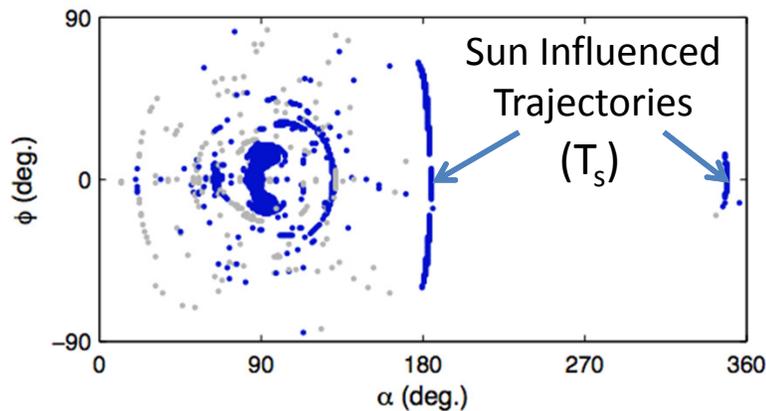


- $\alpha$  defined as before
- $\theta = 0$  (Still captures major regimes of motion)



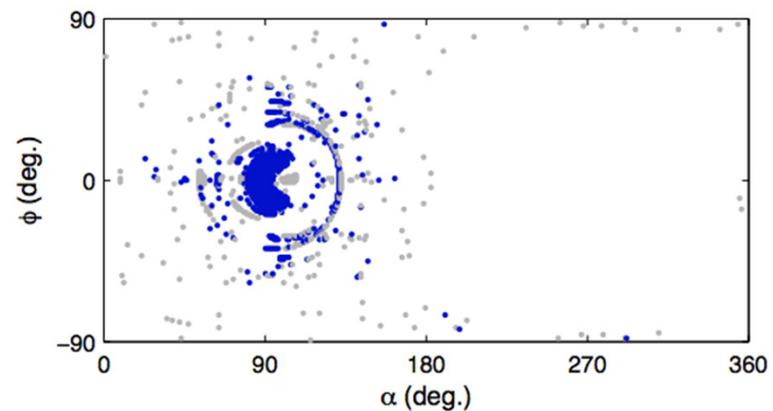
- $\phi$  positive above x-y plane
- Can now pass above or below Earth & Moon

# SEM & EM Spatial Comparison

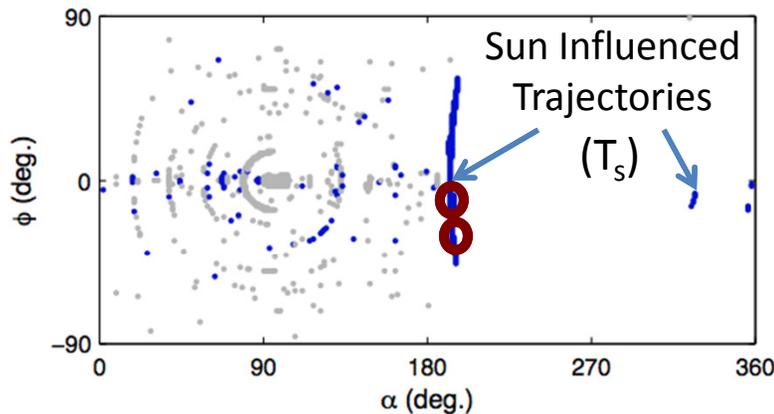


C = 2.4 (SEM)

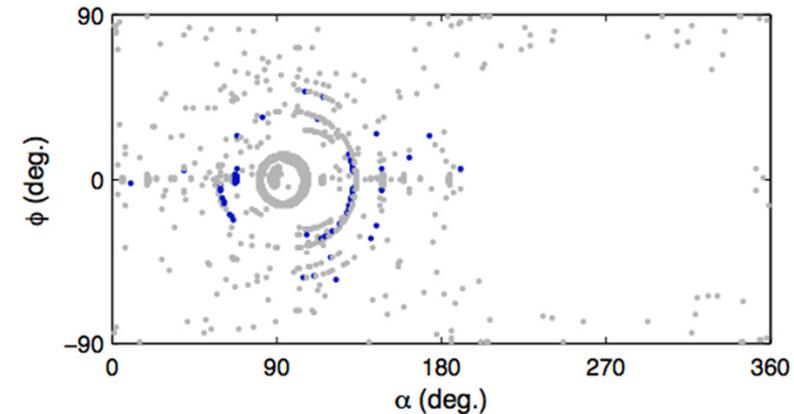
(January 1, 2015)



C = 2.4 (EM)

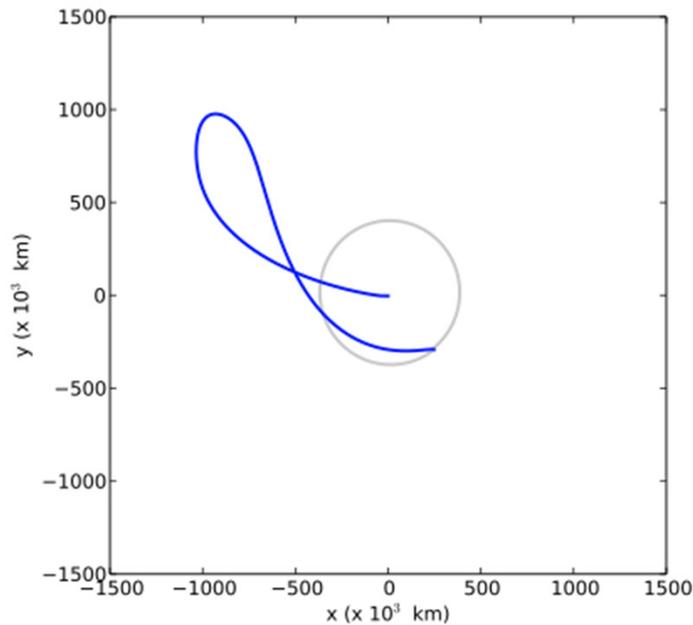


C = 2.6 (SEM)

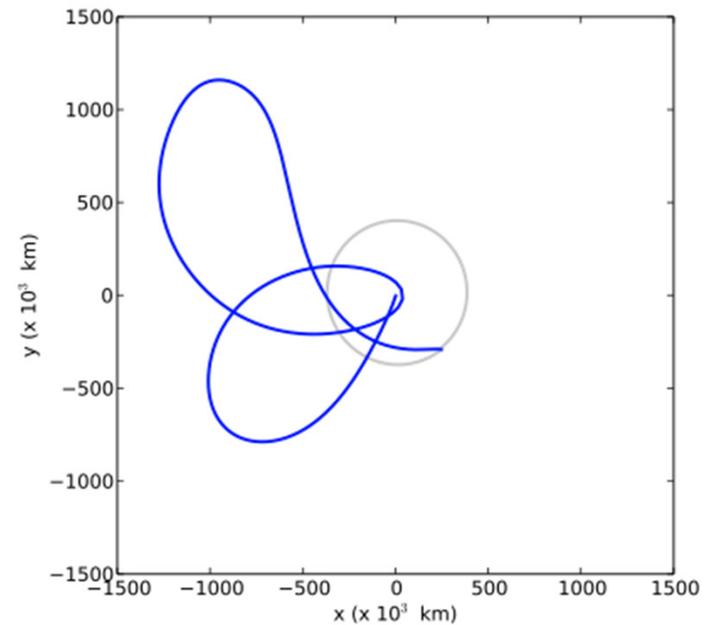


C = 2.6 (EM)

# $T_s$ Trajectories Heuristically Follow Libration Orbit Manifolds



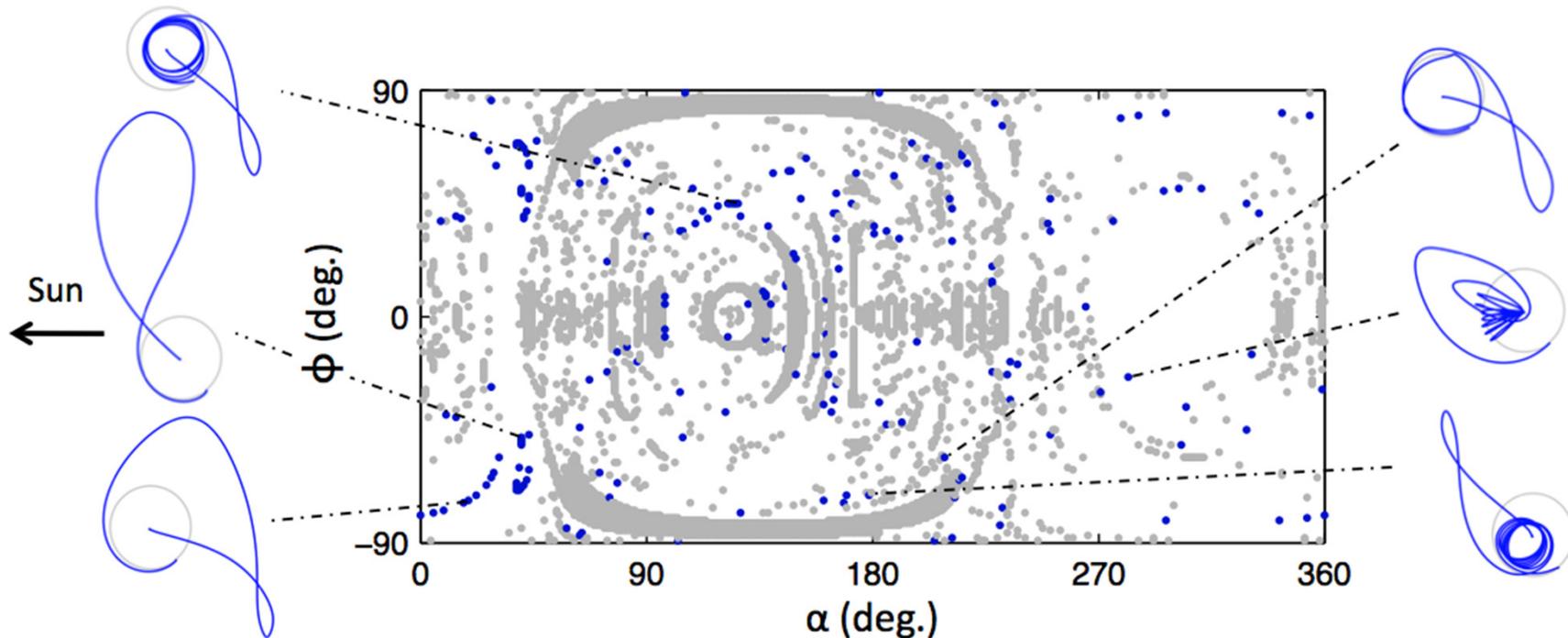
(a)  $C = 2.6$ ,  $\alpha \approx 194.5^\circ$ ,  $\phi \approx -32.0^\circ$



(b)  $C = 2.6$ ,  $\alpha \approx 194.5^\circ$ ,  $\phi \approx -10.0^\circ$

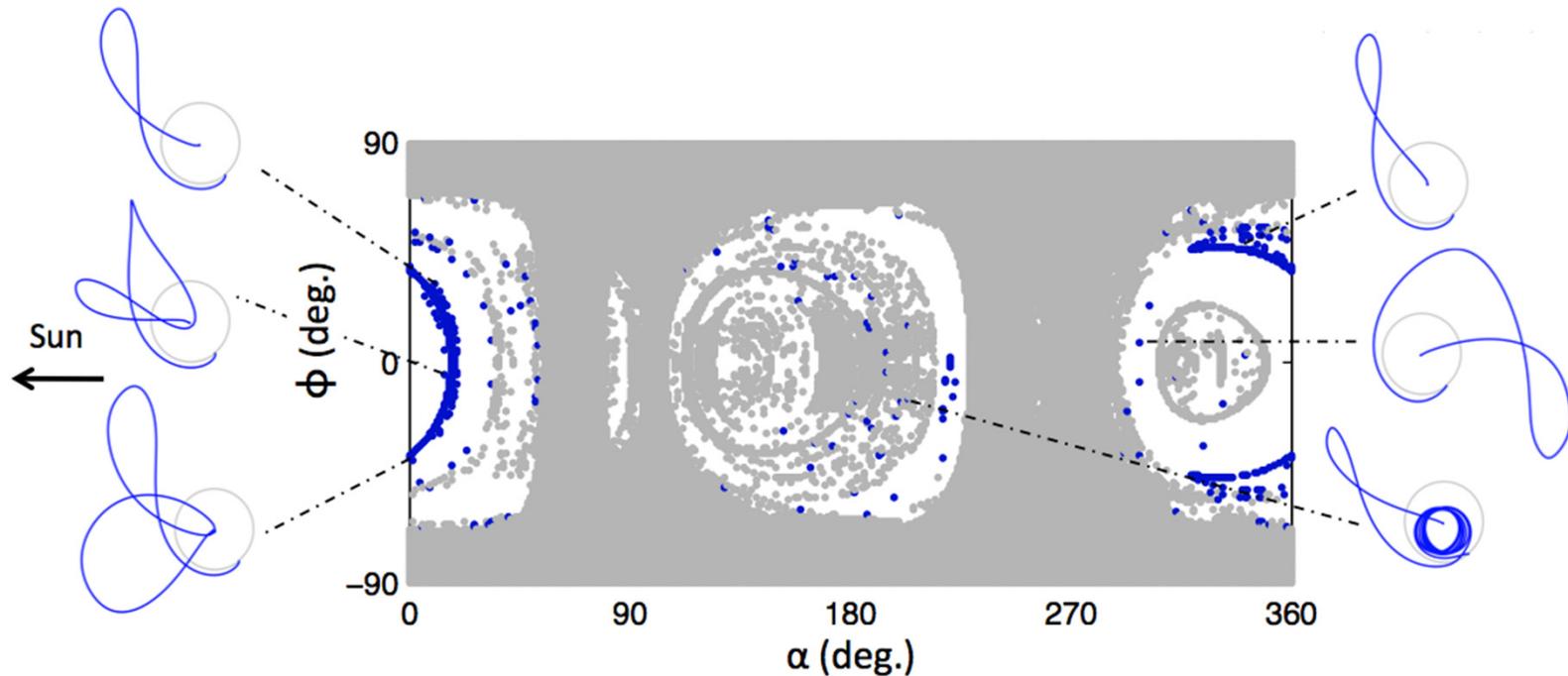
- Two general types of  $T_s$  orbits
- Both travel near Sun-Earth libration points

# Trajectories in SEM at $C = 3.0$



- No trajectories in Earth-Moon system at this  $C$
- Contains trajectories across surface
- Many trajectories with multiple revolutions (higher TOF)

# Trajectories in SEM at $C = 3.1$



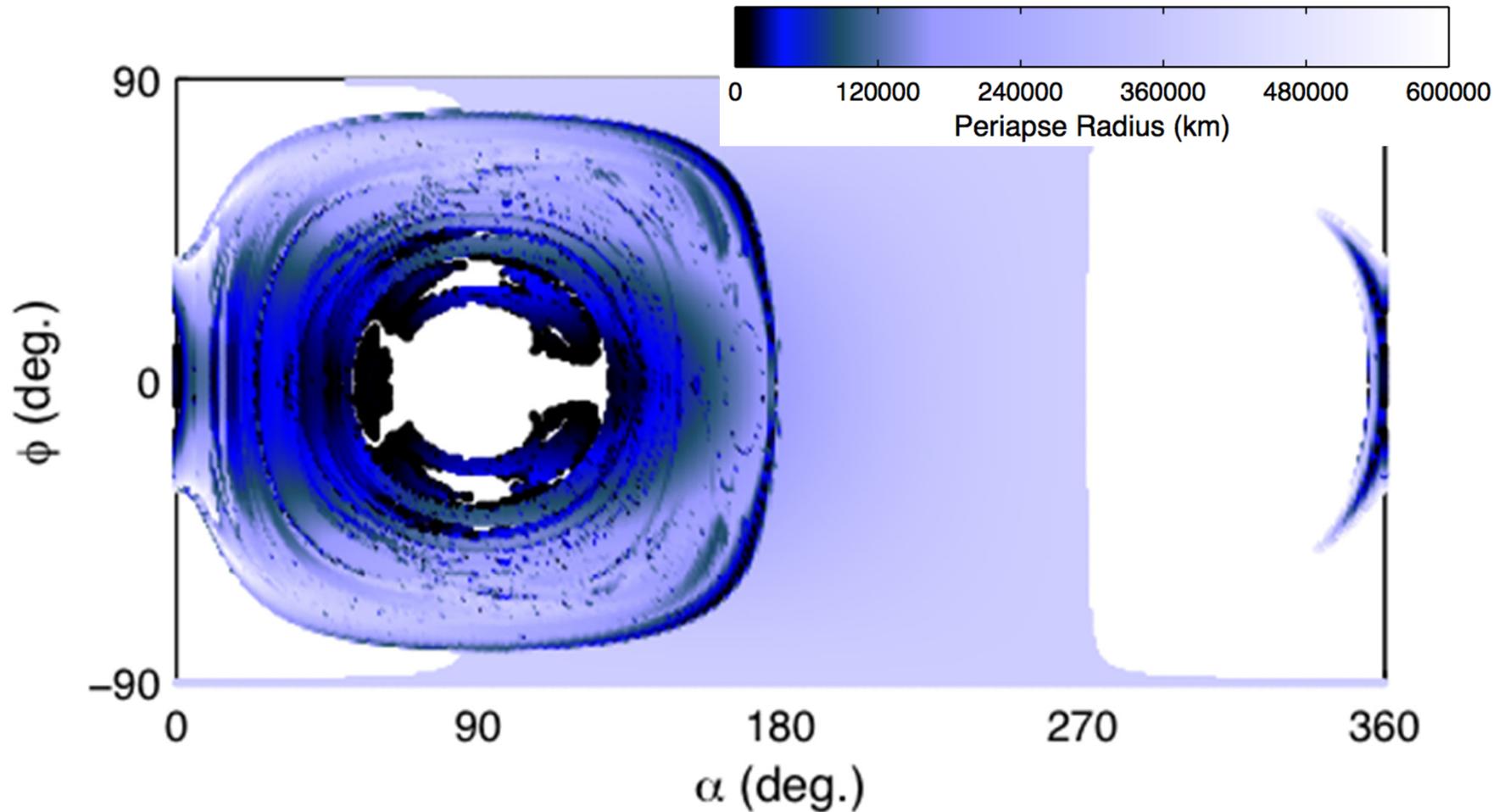
- Distinct families of transfers exist
- Not as spread over surface
- More use Sun-Earth libration dynamics (lower TOF)

# Mission Design Trajectory Characteristics

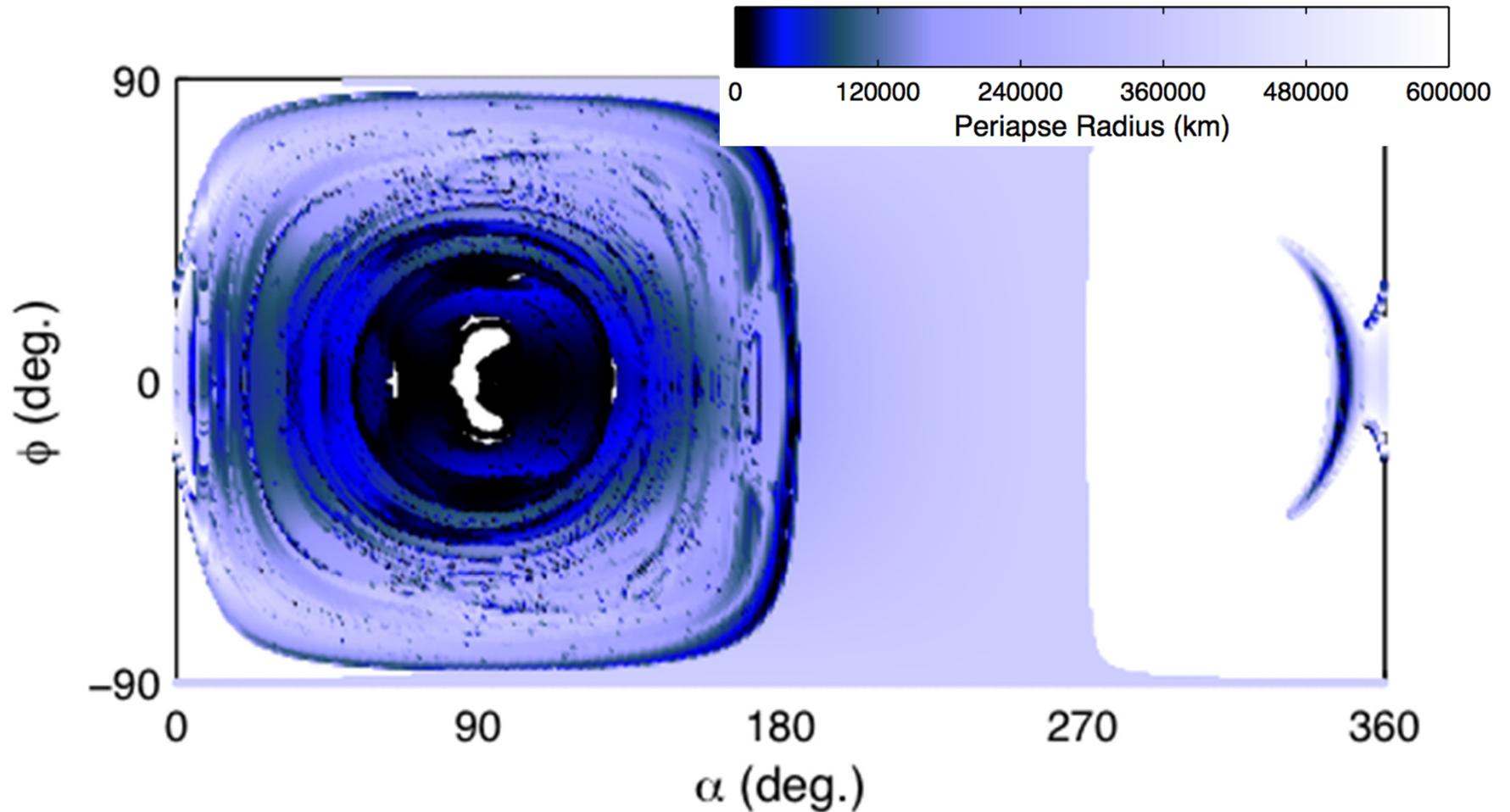
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- Perigee
  - Is a transfer trajectory feasible?
- TOF
  - Is it direct or low-energy?
  - What type of mission is it suitable for?
- $C_3$ 
  - Which launch vehicle is required?
- Earth relative inclination
  - Will it meet the launch constraints?
  - Which launch site is needed?
- Location/velocity at Moon
  - Suitable for lander or impactor?
  - Is landing possible with  $\Delta V$  constraints?

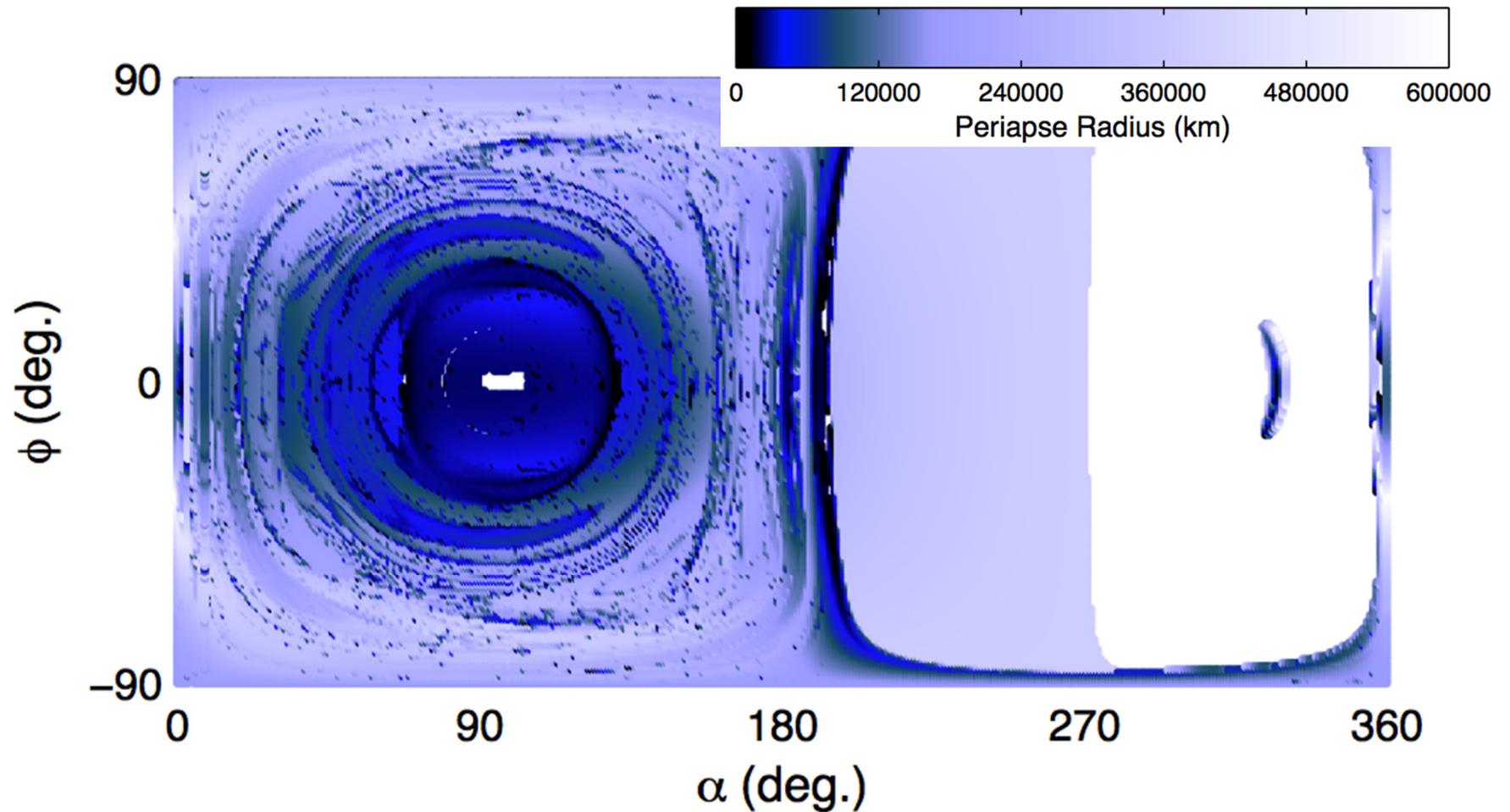
# Lowest Perigee (C = 2.2)



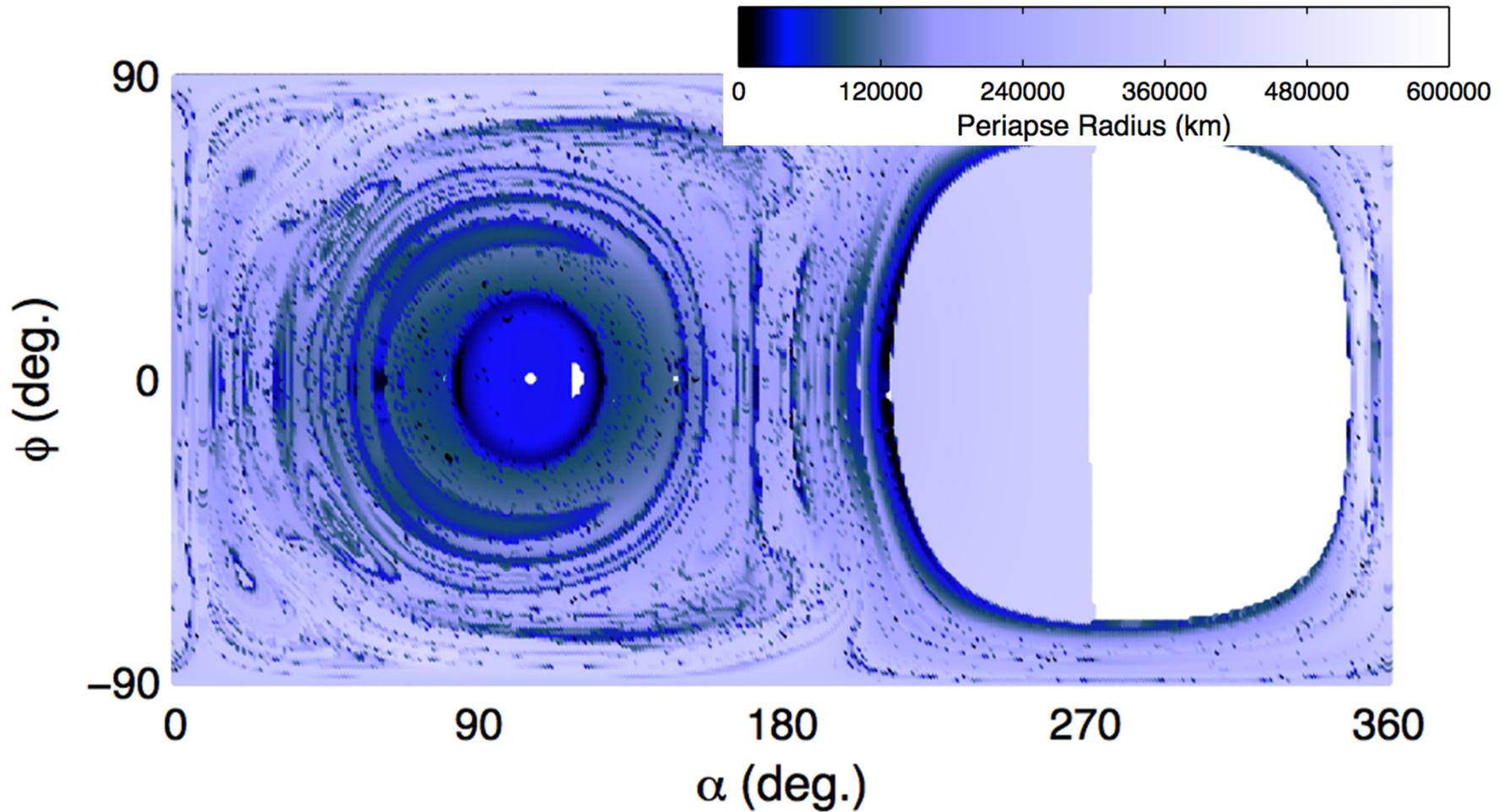
# Lowest Perigee (C = 2.4)



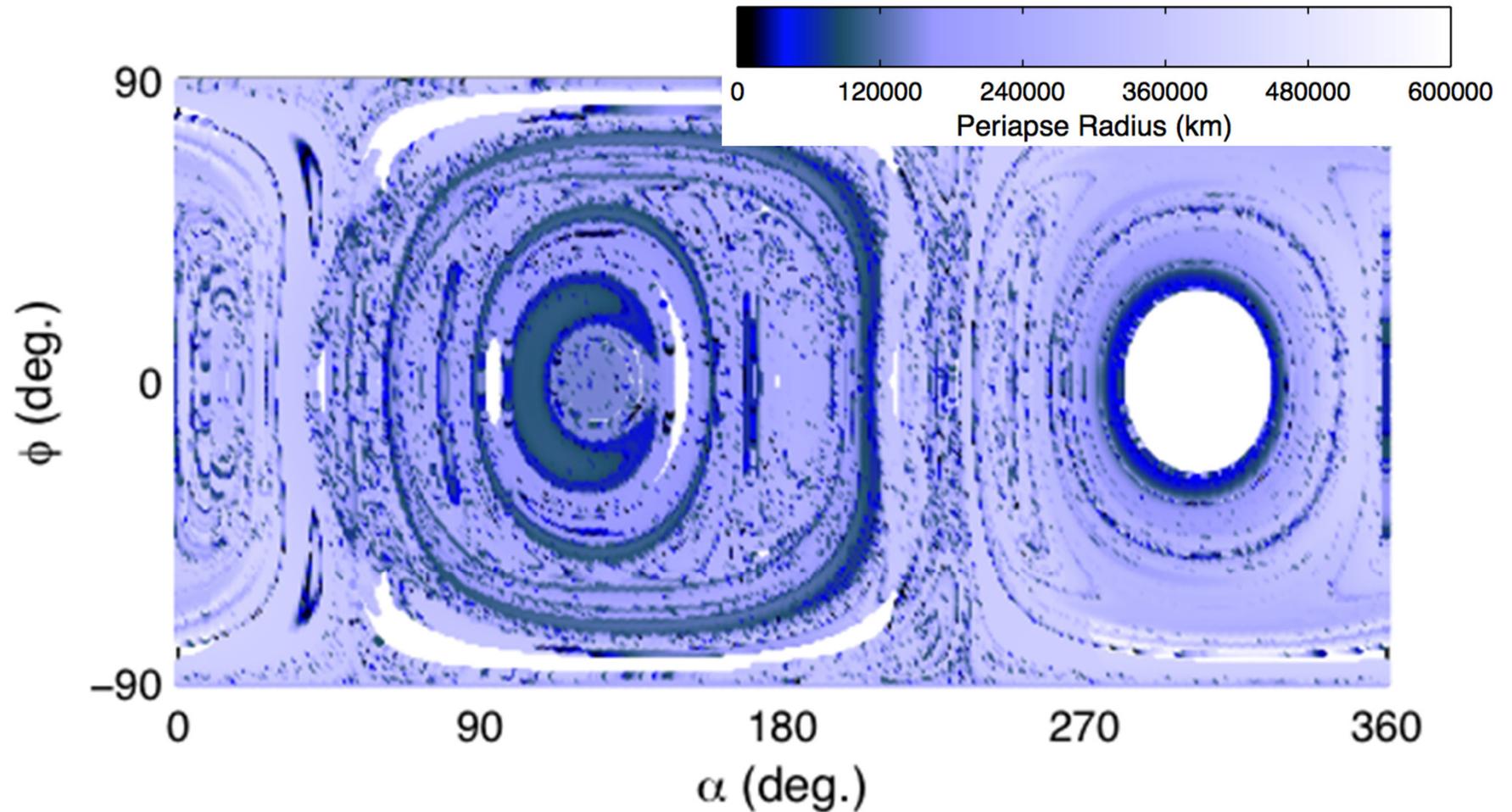
# Lowest Perigee (C = 2.6)



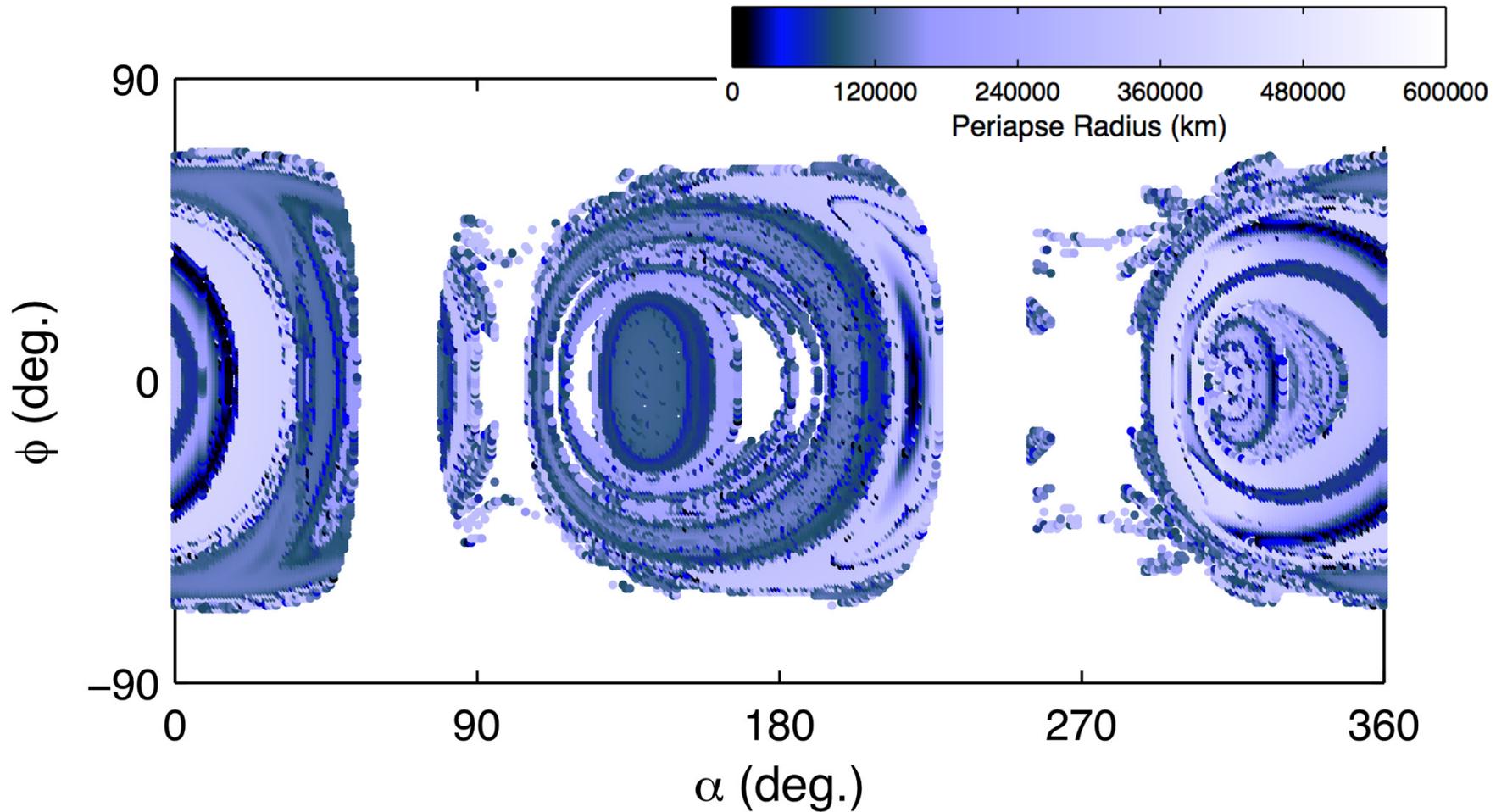
# Lowest Perigee (C = 2.8)



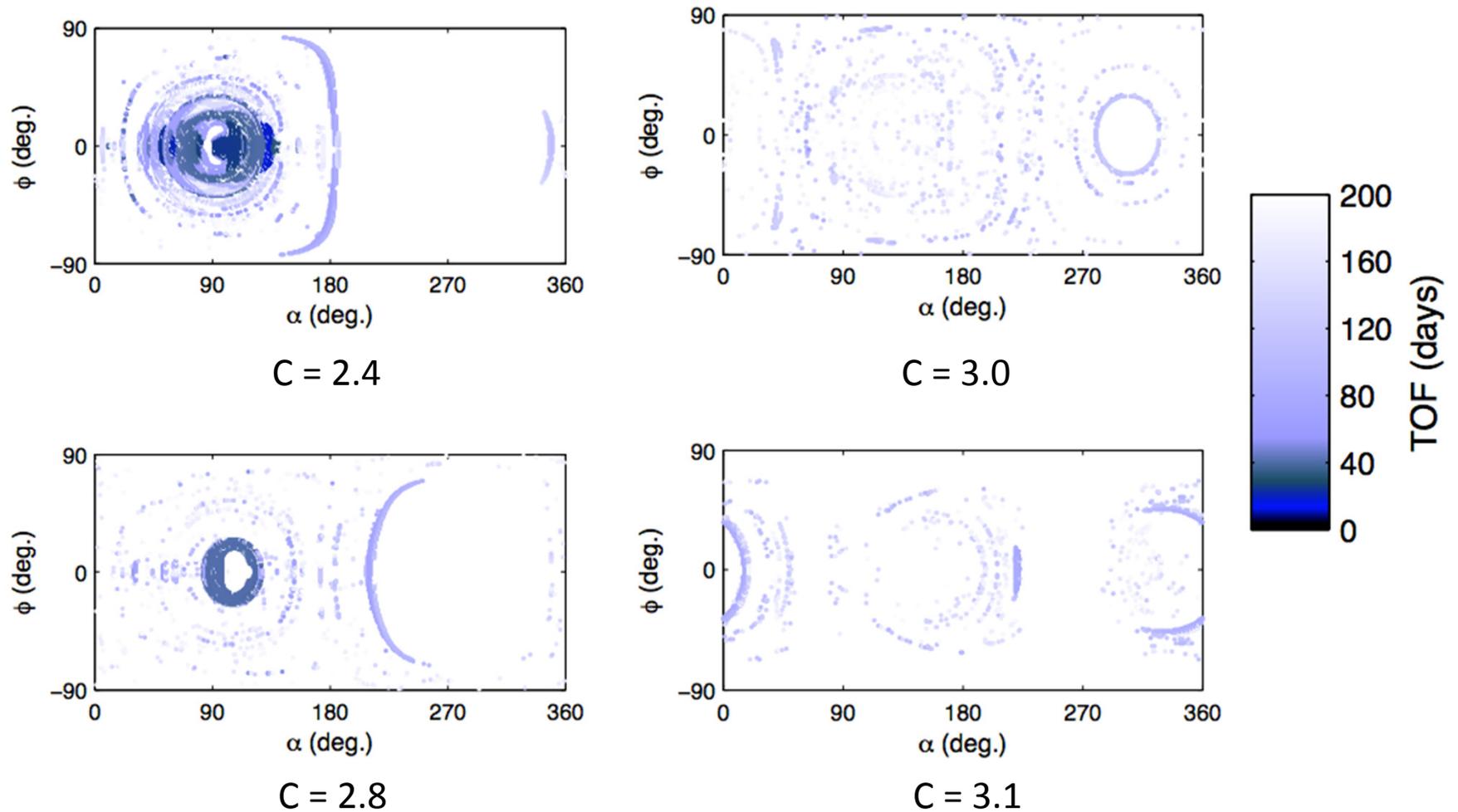
# Lowest Perigee (C = 3.0)



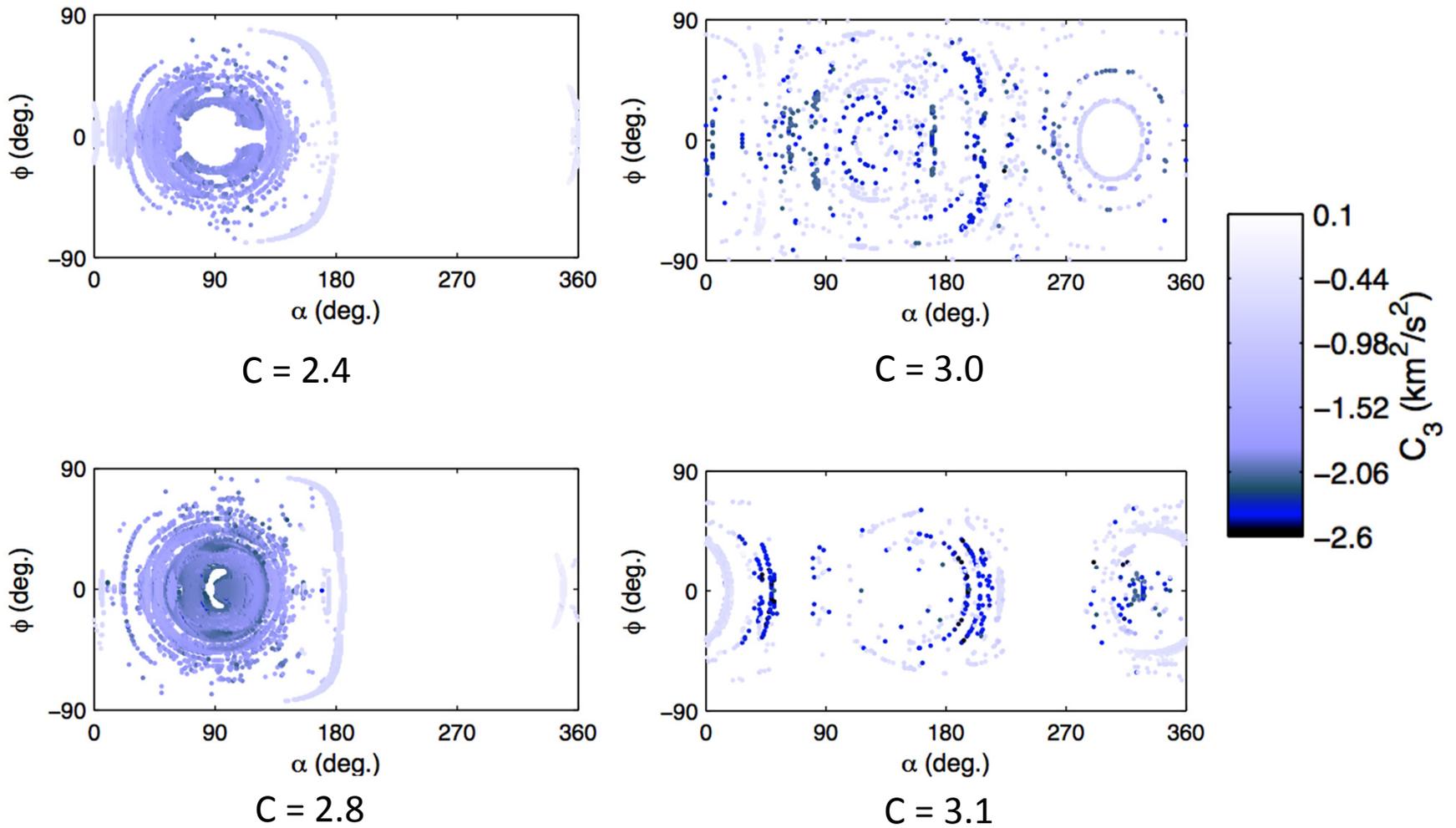
# Lowest Perigee (C = 3.1)



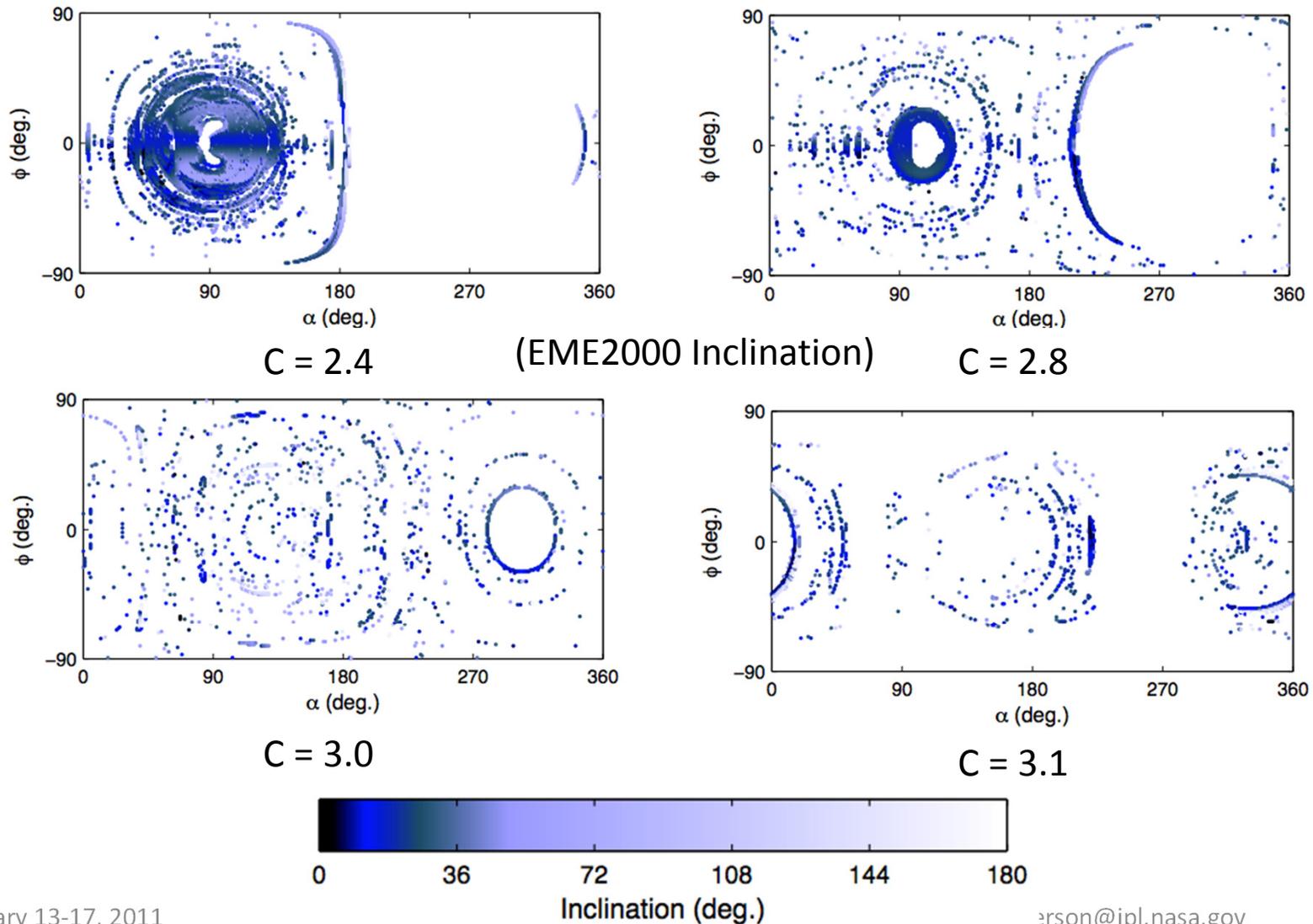
# Time of Flight (TOF)



# Launch Energy ( $C_3$ )



# Many Inclinations Possible Across C



# Conclusions

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- CRTBP approximates Earth-Moon System well
- Statistically significant set of trajectories use Sun
- Sun-influenced trajectories heuristically use libration orbit dynamics
- Higher C
  - Dynamics becomes more chaotic
  - Reach more of surface with variety of different conditions
  - Generally requires greater TOF
- Low-energy trajectories provide many viable trajectory options for different geometry/mission constraints

# Future Work

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- Analyze results for different days/months/years
- Examine various flight path angles
- Survey invariant manifolds across energies and orbit types

# Acknowledgements

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